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THE RECURRENCE TENDENCY AND FORECASTS OF MAGNETIC ACTIVITY

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Abstract--The recurrence tendency in magnetic character figures is evaluated by auto-correlation coefficients for various intervals up to 83 days for each year from 1890 to 1944. The strength of the 27-day recurrence pulse varies during the 55 years, being generally strong from about two years after sunspot maximum through the minimum year. Evidence for persistent recurrence intervals other than multiples of about 27 days is very weak. The magnitudes of the correlation coefficients for short intervals indicate that the time unit of geomagnetic activity is two to three days. Formulas for the prediction of magnetic character figures by the method of least squares are based on the mean auto-correlation coefficients for intervals of 1, 26, 27, 28, and 54 days for stages of the sunspot cycle with weak and strong recurrence conditions. A prediction formula with a non-linear term is also derived.

The recurrence tendency--The tendency for magnetically disturbed or quiet periods to repeat themselves at intervals of about 27, 54 or higher multiples of 27 days is not equally pronounced during the various stages of the solar cycle. Although CHREE and STAGG [see "References" at end of paper, 1927] found the tendency present in each of the years 1906 to 1925, the recurrence was better defined in the years of low sunspot activity and also in the years when the average sunspot latitude was low [see also ARCHENHOLD, 1939]. BARTELS [1932] points out the variability in the strength of the tendency as shown in his charts devised for this purpose. MAUNDER [1905], CHREE and STAGG [1927], and GREAVES and NEWTON [1929] all found that the recurrence tendency was stronger for the moderate types of magnetic disturbances than for the great storms. These results come from analyses by the superposed epoch method of CHREE [1912], in which by selecting disturbed (or quiet) days according to some criterion and computing the average magnetic activity ± n days from the selected day it is possible to show a recurrence tendency without involving a preconception of the length of the interval. Maunder, and Greaves and Newton used the amplitude of disturbance as the basis for selection of days, thereby including many data in years when many disturbances occurred but relatively few data in other years. Chree and Stagg, and Archenhold selected the five most disturbed days in each month, thus equalizing the data from each year, but varying the degree of storminess of the days selected. The former also made separate use of the five quietest days per month, the combination giving perhaps a more uniform body of data. The selection of days of comparable disturbance on the one hand gives an unequal distribution of selected days during the various phases of the cycle of solar or magnetic activity; on the other hand the selection of a uniform number of days each year throughout the cycle causes the degree of disturbance represented by the selected days to vary. Each method presents difficulties in evaluating the time variation of the recurrence tendency.

Evaluation of the recurrence tendency may be accomplished by calculating coefficients of linear correlation between the magnetic character figure for each day and the character figure for the nth succeeding day. Thus

$$r_{0,\,n} = \Sigma x_0 x_n / \sigma_0 \sigma_n$$

where $r_{0,\,n}$, the auto-correlation coefficient between day 0 and n, is derived from the character figures x expressed in units of their standard deviation σ. The recurrence measured in this way will apply for all character figures rather than for some designated degree of disturbance. Year to year comparison of the recurrence tendency is effected by calculation of the coefficients separately for each year. Auto-correlation coefficients showing this recurrence were computed for the International Magnetic Character Figures, C, which are listed by CHAPMAN and BARTELS [1940] for 1890 to 1937 and tabulated periodically in the Journal of Terrestrial Magnetism and Atmospheric Electricity for 1938 to 1944. The index C rates every day on a scale of increasing magnetic disturbance from 0.0 to 2.0, a range convenient for correlation analyses. The coefficients were computed for each year separately; thus each group has a sufficiently large population that, although the distribution of C-figures is not normal and varies slightly from year to year, regrouping seemed unnecesary. The computations were carried out by the Mendenhall-Warren-Hollerith correlation method, [WARREN and MENDENHALL, 1929], an exact method employing punch card

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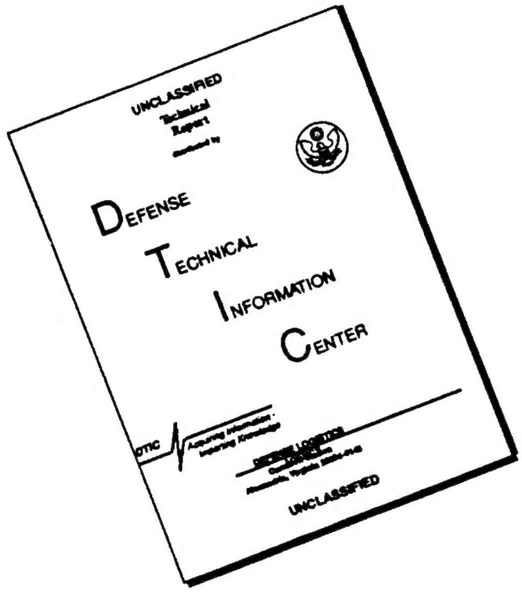

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Table 1--Auto-correlation coefficients from magnetic character

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
1890	49	17	05	06	07	07	09	03	$\overline{02}$	$\overline{12}$	$\overline{08}$	$\overline{06}$	$\overline{05}$	$\overline{05}$	$\overline{09}$	$\overline{11}$	$\overline{04}$	..	$\overline{02}$	04	01	..	$\overline{10}$
91	53	18	02	$\overline{08}$	$\overline{02}$	$\overline{04}$	$\overline{06}$	$\overline{07}$	$\overline{11}$	$\overline{05}$	$\overline{01}$	$\overline{01}$	$\overline{02}$	$\overline{05}$	$\overline{05}$	04	03	..	$\overline{04}$	$\overline{02}$	07	..	$\overline{03}$
92	49	16	03	03	05	$\overline{02}$	$\overline{10}$	$\overline{12}$	$\overline{10}$	$\overline{11}$	$\overline{07}$	06	08	07	05	05	10	..	$\overline{03}$	$\overline{03}$	$\overline{02}$	..	$\overline{04}$
93	49	19	03	$\overline{11}$	$\overline{14}$	$\overline{18}$	$\overline{10}$	$\overline{11}$	00	03	04	02	$\overline{07}$	$\overline{03}$	$\overline{06}$	$\overline{09}$	$\overline{08}$	..	$\overline{12}$	$\overline{15}$	$\overline{08}$	..	$\overline{03}$
94	48	14	01	06	05	01	04	01	$\overline{06}$	$\overline{11}$	$\overline{11}$	$\overline{13}$	$\overline{07}$	$\overline{11}$	$\overline{12}$	$\overline{09}$	$\overline{10}$	..	$\overline{07}$	$\overline{08}$	$\overline{05}$	..	$\overline{06}$
95	50	15	05	07	03	$\overline{05}$	$\overline{07}$	$\overline{04}$	$\overline{08}$	$\overline{06}$	$\overline{02}$	12	22	27	17	03	$\overline{01}$	..	$\overline{10}$	$\overline{10}$	$\overline{07}$	..	$\overline{08}$
96	54	27	10	00	$\overline{05}$	$\overline{09}$	$\overline{10}$	$\overline{05}$	$\overline{03}$	$\overline{03}$	$\overline{01}$	$\overline{05}$	$\overline{03}$	$\overline{01}$	00	$\overline{05}$	$\overline{08}$	..	$\overline{01}$	01	01	..	07
97	44	08	02	04	$\overline{01}$	$\overline{05}$	$\overline{04}$	$\overline{03}$	07	05	$\overline{01}$	$\overline{07}$	$\overline{11}$	$\overline{10}$	$\overline{07}$	05	04	..	$\overline{02}$	$\overline{03}$	$\overline{05}$	..	$\overline{06}$
98	50	17	12	07	01	$\overline{04}$	$\overline{09}$	$\overline{08}$	$\overline{17}$	$\overline{18}$	$\overline{15}$	$\overline{18}$	$\overline{16}$	$\overline{12}$	$\overline{13}$	$\overline{16}$	$\overline{14}$	..	$\overline{04}$	$\overline{10}$	$\overline{12}$	..	08
99	55	20	05	$\overline{06}$	$\overline{11}$	$\overline{13}$	$\overline{05}$	01	06	10	16	07	$\overline{01}$	05	03	$\overline{01}$	$\overline{01}$	..	$\overline{02}$	$\overline{12}$	$\overline{16}$	..	$\overline{10}$
1900	39	00	$\overline{04}$	10	22	24	20	02	$\overline{08}$	$\overline{05}$	$\overline{02}$	04	02	$\overline{01}$	$\overline{07}$	01	07	..	06	11	12	..	03
01	34	$\overline{01}$	$\overline{08}$	$\overline{13}$	$\overline{02}$	00	01	$\overline{04}$	$\overline{06}$	$\overline{07}$	$\overline{10}$	$\overline{02}$	00	$\overline{04}$	$\overline{05}$	$\overline{06}$	$\overline{02}$	..	$\overline{08}$	06	08	..	$\overline{07}$
02	42	07	$\overline{07}$	$\overline{12}$	$\overline{19}$	$\overline{17}$	$\overline{11}$	$\overline{08}$	$\overline{05}$	01	$\overline{05}$	$\overline{04}$	$\overline{01}$	$\overline{02}$	$\overline{07}$	02	02	..	00	$\overline{13}$	$\overline{16}$	..	$\overline{04}$
03	48	12	04	00	$\overline{03}$	02	04	08	06	06	05	05	02	$\overline{02}$	02	06	16	..	12	06	04	..	00
04	43	04	$\overline{11}$	$\overline{13}$	$\overline{08}$	01	04	00	02	$\overline{05}$	$\overline{03}$	$\overline{01}$	$\overline{02}$	01	$\overline{04}$	$\overline{03}$	$\overline{04}$	..	$\overline{02}$	05	01	..	$\overline{06}$
05	48	15	01	$\overline{02}$	01	00	02	02	08	04	02	$\overline{02}$	$\overline{08}$	$\overline{05}$	$\overline{03}$	$\overline{02}$	02	..	02	04	01	..	$\overline{09}$
06	48	07	$\overline{07}$	$\overline{04}$	01	03	05	06	04	$\overline{05}$	$\overline{10}$	$\overline{10}$	$\overline{04}$	$\overline{01}$	$\overline{02}$	$\overline{01}$	05	..	11	10	$\overline{02}$	..	$\overline{12}$
07	47	09	$\overline{03}$	$\overline{13}$	$\overline{09}$	$\overline{03}$	$\overline{06}$	$\overline{10}$	$\overline{06}$	$\overline{07}$	$\overline{05}$	$\overline{03}$	12	09	05	04	$\overline{07}$	..	$\overline{07}$	$\overline{03}$	$\overline{03}$	..	$\overline{08}$
08	57	19	$\overline{02}$	$\overline{09}$	$\overline{04}$	07	07	06	03	00	$\overline{11}$	$\overline{13}$	$\overline{14}$	$\overline{17}$	$\overline{21}$	$\overline{12}$	$\overline{06}$	..	11	00	$\overline{03}$	..	$\overline{03}$
09	52	19	07	14	00	$\overline{04}$	$\overline{03}$	$\overline{09}$	$\overline{12}$	$\overline{17}$	$\overline{18}$	$\overline{14}$	$\overline{14}$	$\overline{11}$	$\overline{08}$	$\overline{07}$	$\overline{10}$	..	$\overline{15}$	$\overline{08}$	$\overline{02}$	..	07
1910	52	19	05	$\overline{04}$	$\overline{05}$	$\overline{05}$	$\overline{01}$	01	00	$\overline{02}$	01	03	02	07	06	02	$\overline{05}$	..	$\overline{11}$	$\overline{06}$	$\overline{04}$	..	$\overline{02}$
11	50	12	02	05	13	09	07	05	$\overline{01}$	01	00	$\overline{10}$	$\overline{18}$	$\overline{14}$	$\overline{07}$	$\overline{01}$	$\overline{02}$	..	07	17	15	..	08
12	43	01	$\overline{12}$	$\overline{08}$	04	$\overline{01}$	$\overline{04}$	$\overline{11}$	$\overline{10}$	$\overline{07}$	$\overline{04}$	$\overline{04}$	$\overline{09}$	$\overline{15}$	$\overline{12}$	$\overline{07}$	$\overline{03}$	..	$\overline{09}$	02	10	..	$\overline{05}$
13	51	22	06	$\overline{04}$	01	$\overline{05}$	$\overline{06}$	$\overline{11}$	$\overline{11}$	$\overline{11}$	$\overline{10}$	$\overline{08}$	$\overline{06}$	$\overline{06}$	$\overline{10}$	$\overline{11}$	$\overline{07}$	..	$\overline{07}$	$\overline{05}$	$\overline{05}$	..	01
14	42	09	$\overline{04}$	$\overline{02}$	04	06	05	04	$\overline{02}$	$\overline{03}$	02	$\overline{01}$	$\overline{01}$	09	09	$\overline{13}$	$\overline{07}$	..	$\overline{03}$	06	01	..	$\overline{04}$
15	52	14	10	12	05	$\overline{07}$	$\overline{11}$	$\overline{08}$	$\overline{05}$	$\overline{01}$	00	$\overline{04}$	$\overline{05}$	$\overline{02}$	01	03	01	..	$\overline{09}$	$\overline{10}$	$\overline{06}$	..	03
16	51	12	$\overline{01}$	$\overline{04}$	$\overline{05}$	$\overline{07}$	$\overline{02}$	$\overline{06}$	$\overline{06}$	09	$\overline{09}$	$\overline{03}$	07	11	03	$\overline{08}$	$\overline{14}$	..	$\overline{04}$	04	03	..	$\overline{09}$
17	47	12	$\overline{04}$	$\overline{15}$	$\overline{15}$	$\overline{13}$	$\overline{08}$	$\overline{02}$	03	02	$\overline{01}$	02	06	07	06	04	04	..	00	$\overline{01}$	$\overline{09}$	..	$\overline{14}$
18	56	17	02	$\overline{03}$	$\overline{07}$	$\overline{10}$	$\overline{11}$	$\overline{08}$	$\overline{14}$	$\overline{12}$	$\overline{04}$	02	00	01	05	07	02	..	$\overline{06}$	$\overline{12}$	$\overline{11}$	..	04
19	47	07	$\overline{07}$	$\overline{09}$	$\overline{12}$	$\overline{13}$	$\overline{04}$	$\overline{07}$	$\overline{03}$	$\overline{03}$	03	05	04	08	08	08	03	..	02	02	04	..	$\overline{11}$
1920	44	10	$\overline{11}$	$\overline{15}$	$\overline{12}$	$\overline{04}$	$\overline{07}$	01	09	04	00	05	02	00	$\overline{07}$	$\overline{07}$	$\overline{01}$	..	02	$\overline{01}$	$\overline{02}$	..	00
21	50	15	12	04	06	04	$\overline{02}$	$\overline{09}$	$\overline{06}$	$\overline{04}$	$\overline{11}$	$\overline{09}$	$\overline{07}$	$\overline{11}$	$\overline{05}$	$\overline{01}$	$\overline{11}$	..	$\overline{04}$	02	$\overline{04}$	..	04
22	54	22	02	$\overline{09}$	$\overline{20}$	$\overline{20}$	$\overline{09}$	$\overline{09}$	$\overline{07}$	03	08	17	17	21	20	11	04	..	$\overline{11}$	$\overline{13}$	$\overline{14}$	..	$\overline{10}$
23	54	15	$\overline{06}$	$\overline{15}$	$\overline{21}$	$\overline{20}$	$\overline{12}$	$\overline{08}$	00	02	01	$\overline{06}$	$\overline{05}$	$\overline{02}$	08	12	17	..	$\overline{02}$	$\overline{09}$	$\overline{07}$	..	$\overline{13}$
24	46	14	$\overline{01}$	$\overline{05}$	$\overline{08}$	$\overline{03}$	02	$\overline{05}$	$\overline{06}$	$\overline{08}$	$\overline{06}$	$\overline{03}$	$\overline{05}$	$\overline{05}$	$\overline{10}$	$\overline{09}$	$\overline{08}$	..	05	$\overline{03}$	$\overline{03}$	..	$\overline{09}$
25	45	07	07	03	$\overline{01}$	$\overline{10}$	$\overline{06}$	$\overline{05}$	00	04	01	02	04	06	03	00	02	..	$\overline{01}$	$\overline{01}$	$\overline{01}$	..	09
26	54	19	09	08	05	05	10	11	04	$\overline{01}$	00	06	01	$\overline{06}$	$\overline{06}$	$\overline{07}$	$\overline{11}$	..	$\overline{01}$	$\overline{01}$	00	..	04
27	53	21	06	03	$\overline{01}$	$\overline{07}$	$\overline{11}$	$\overline{10}$	02	09	06	06	04	04	07	05	02	..	$\overline{04}$	$\overline{05}$	$\overline{06}$	..	$\overline{09}$
28	54	19	06	$\overline{03}$	02	02	$\overline{04}$	$\overline{04}$	$\overline{11}$	$\overline{06}$	00	$\overline{04}$	$\overline{05}$	$\overline{02}$	03	06	01	..	$\overline{01}$	$\overline{02}$	$\overline{01}$	..	10
29	55	17	05	05	05	$\overline{01}$	$\overline{09}$	$\overline{08}$	$\overline{06}$	02	08	12	14	09	04	03	00	..	$\overline{11}$	$\overline{10}$	$\overline{08}$	..	$\overline{09}$
1930	62	25	08	$\overline{04}$	$\overline{14}$	$\overline{20}$	$\overline{17}$	$\overline{18}$	02	06	05	16	17	14	14	09	01	..	$\overline{12}$	$\overline{15}$	$\overline{16}$	..	$\overline{09}$
31	55	19	$\overline{04}$	$\overline{14}$	$\overline{07}$	$\overline{03}$	01	09	07	10	14	05	05	02	06	13	17	..	08	$\overline{07}$	$\overline{11}$	..	$\overline{06}$
32	60	27	05	$\overline{08}$	$\overline{14}$	$\overline{16}$	$\overline{14}$	$\overline{07}$	02	05	05	02	$\overline{02}$	$\overline{02}$	04	04	04	..	$\overline{09}$	$\overline{11}$	$\overline{14}$	..	00
33	60	29	12	07	04	$\overline{03}$	$\overline{05}$	$\overline{12}$	$\overline{18}$	$\overline{18}$	$\overline{18}$	$\overline{05}$	00	$\overline{01}$	$\overline{08}$	$\overline{11}$	$\overline{18}$	..	$\overline{12}$	$\overline{11}$	$\overline{07}$	..	$\overline{03}$
34	46	15	07	03	02	05	$\overline{01}$	$\overline{04}$	$\overline{07}$	$\overline{04}$	$\overline{09}$	$\overline{14}$	$\overline{06}$	$\overline{06}$	$\overline{06}$	$\overline{05}$	00	..	$\overline{01}$	02	05	..	02
35	51	19	06	$\overline{04}$	$\overline{03}$	00	$\overline{03}$	$\overline{04}$	$\overline{04}$	00	02	$\overline{02}$	05	00	$\overline{01}$	00	$\overline{04}$	..	03	$\overline{04}$	$\overline{04}$	..	$\overline{06}$
36	48	20	09	02	01	01	03	$\overline{02}$	$\overline{06}$	$\overline{03}$	$\overline{07}$	$\overline{10}$	$\overline{14}$	$\overline{08}$	$\overline{06}$	$\overline{08}$	$\overline{01}$	..	$\overline{08}$	$\overline{07}$	02	..	10
37	48	15	09	07	00	$\overline{08}$	$\overline{04}$	$\overline{03}$	04	00	00	02	05	$\overline{01}$	$\overline{02}$	$\overline{02}$	$\overline{06}$	..	$\overline{11}$	$\overline{09}$	$\overline{12}$	..	$\overline{05}$
38	54	26	12	03	$\overline{01}$	$\overline{03}$	$\overline{06}$	$\overline{06}$	$\overline{09}$	$\overline{09}$	$\overline{09}$	$\overline{10}$	$\overline{16}$	$\overline{12}$	$\overline{07}$	$\overline{04}$	01	..	09	05	00	..	19
39	52	16	03	00	08	11	06	02	$\overline{11}$	$\overline{10}$	$\overline{01}$	06	08	08	00	$\overline{07}$	$\overline{04}$	..	$\overline{02}$	$\overline{02}$	02	..	$\overline{02}$
1940	54	18	07	08	07	04	00	00	02	02	$\overline{05}$	$\overline{09}$	$\overline{19}$	$\overline{27}$	$\overline{27}$	$\overline{20}$	$\overline{14}$	..	$\overline{05}$	$\overline{03}$	$\overline{05}$	..	$\overline{09}$
41	50	18	00	$\overline{08}$	$\overline{08}$	$\overline{06}$	$\overline{06}$	$\overline{02}$	$\overline{02}$	$\overline{01}$	$\overline{03}$	$\overline{06}$	$\overline{07}$	$\overline{04}$	$\overline{05}$	03	09	..	07	09	07	..	00
42	49	18	11	07	06	$\overline{03}$	$\overline{09}$	$\overline{08}$	00	02	$\overline{03}$	09	18	15	08	11	01	..	$\overline{12}$	$\overline{13}$	$\overline{06}$	..	$\overline{05}$
43	63	33	16	15	09	01	$\overline{12}$	$\overline{16}$	$\overline{21}$	$\overline{13}$	$\overline{05}$	00	$\overline{02}$	$\overline{04}$	00	$\overline{09}$	$\overline{09}$	..	$\overline{14}$	09	$\overline{02}$	..	11
44	50	24	23	14	12	13	01	$\overline{05}$	$\overline{07}$	$\overline{01}$	$\overline{08}$	$\overline{05}$	01	01	$\overline{02}$	$\overline{10}$	$\overline{13}$	..	$\overline{06}$	03	10	..	16

figures (multiplied by 100; negative values overscored)

24	25	26	27	28	29	30	31	32	33	34	35	36	52	53	54	55	56	79	80	81	82	83
$\overline{03}$	05	18	28	35	28	18	09	09	11	09	04	04	$\overline{11}$	$\overline{10}$	01	03	11	$\overline{15}$	$\overline{11}$	$\overline{10}$	$\overline{10}$	$\overline{03}$
00	04	12	20	23	19	08	$\overline{01}$	00	$\overline{03}$	$\overline{11}$	$\overline{15}$	$\overline{07}$	$\overline{02}$	$\overline{06}$	$\overline{10}$	$\overline{06}$	$\overline{02}$	$\overline{03}$	$\overline{08}$	$\overline{12}$	$\overline{11}$	07
$\overline{07}$	$\overline{02}$	$\overline{01}$	05	04	07	04	$\overline{03}$	$\overline{07}$	$\overline{11}$	$\overline{09}$	$\overline{13}$	$\overline{15}$	00	07	16	17	15	07	05	12	19	12
05	10	15	22	16	12	05	$\overline{03}$	$\overline{03}$	$\overline{03}$	$\overline{04}$	$\overline{03}$	01	08	12	09	11	11	04	12	14	12	00
$\overline{05}$	06	13	11	04	07	12	09	02	$\overline{02}$	$\overline{02}$	$\overline{05}$	$\overline{10}$	07	11	16	17	13	$\overline{02}$	$\overline{04}$	$\overline{03}$	00	03
$\overline{01}$	12	28	38	33	21	10	00	$\overline{06}$	$\overline{10}$	$\overline{10}$	$\overline{07}$	$\overline{07}$	06	15	26	28	17	10	16	17	16	20
13	23	35	44	43	31	12	$\overline{06}$	$\overline{10}$	$\overline{13}$	$\overline{16}$	$\overline{11}$	$\overline{01}$	18	25	23	27	21	12	10	13	13	07
00	09	22	27	19	04	06	02	00	01	02	03	05	03	19	22	12	05	15	18	13	$\overline{05}$	$\overline{11}$
13	16	30	44	37	22	14	08	05	$\overline{04}$	$\overline{07}$	$\overline{08}$	$\overline{16}$	15	20	26	23	14	09	12	12	16	15
$\overline{05}$	10	28	39	28	12	00	$\overline{10}$	$\overline{12}$	$\overline{14}$	$\overline{14}$	$\overline{12}$	$\overline{02}$	10	20	31	26	17	04	12	18	14	10
00	14	24	18	12	06	06	05	10	11	05	01	$\overline{01}$	12	23	20	10	07	00	11	09	03	$\overline{02}$
$\overline{17}$	$\overline{08}$	17	28	25	22	00	$\overline{06}$	$\overline{07}$	04	10	$\overline{01}$	$\overline{15}$	$\overline{07}$	06	20	25	06	$\overline{02}$	10	22	20	11
$\overline{03}$	$\overline{01}$	10	20	24	16	08	03	00	$\overline{07}$	$\overline{11}$	$\overline{15}$	$\overline{11}$	$\overline{12}$	$\overline{07}$	00	10	16	$\overline{08}$	$\overline{10}$	02	04	03
06	13	20	26	27	16	11	01	05	18	09	01	05	05	12	18	18	15	07	08	12	10	08
$\overline{04}$	00	07	12	08	06	$\overline{02}$	$\overline{05}$	$\overline{02}$	00	04	00	$\overline{09}$	$\overline{02}$	03	02	$\overline{02}$	$\overline{02}$	03	05	06	$\overline{04}$	$\overline{11}$
$\overline{08}$	01	08	20	18	14	19	08	06	01	$\overline{11}$	$\overline{12}$	$\overline{09}$	$\overline{08}$	$\overline{04}$	00	09	07	$\overline{10}$	$\overline{06}$	$\overline{02}$	05	06
$\overline{05}$	01	09	17	13	03	$\overline{08}$	$\overline{13}$	$\overline{14}$	$\overline{05}$	02	$\overline{01}$	$\overline{04}$	03	08	12	14	08	13	11	09	01	03
$\overline{04}$	09	17	17	10	06	08	03	$\overline{05}$	$\overline{04}$	$\overline{06}$	$\overline{08}$	$\overline{11}$	00	08	09	08	04	00	01	$\overline{01}$	01	02
07	17	29	29	20	08	02	00	05	07	05	02	$\overline{01}$	14	16	12	09	09	16	19	16	07	08
04	$\overline{02}$	10	20	22	13	09	02	02	01	03	$\overline{01}$	$\overline{04}$	08	10	14	08	08	$\overline{01}$	00	$\overline{02}$	$\overline{02}$	04
$\overline{02}$	06	24	32	24	15	08	05	$\overline{03}$	$\overline{03}$	01	03	07	00	08	15	14	09	$\overline{15}$	$\overline{09}$	02	08	06
09	19	35	46	37	16	06	05	06	08	06	02	00	13	31	41	28	10	01	13	24	26	09
$\overline{09}$	04	25	33	17	02	$\overline{06}$	$\overline{06}$	02	09	04	$\overline{08}$	$\overline{13}$	10	26	29	14	03	04	14	12	03	$\overline{02}$
07	21	34	38	32	21	10	07	02	$\overline{08}$	$\overline{09}$	$\overline{10}$	$\overline{14}$	01	11	14	16	09	07	10	08	01	$\overline{09}$
$\overline{01}$	06	04	04	10	14	10	13	10	02	02	10	$\overline{02}$	$\overline{04}$	00	04	06	$\overline{02}$	$\overline{03}$	02	$\overline{08}$	$\overline{02}$	03
10	13	20	30	27	17	10	09	00	$\overline{11}$	$\overline{12}$	$\overline{10}$	$\overline{08}$	08	09	14	18	16	00	03	03	06	06
$\overline{10}$	$\overline{08}$	00	12	09	01	$\overline{09}$	$\overline{06}$	$\overline{02}$	$\overline{04}$	$\overline{01}$	04	00	07	15	15	13	11	$\overline{04}$	04	04	01	$\overline{03}$
$\overline{08}$	05	14	13	07	05	$\overline{04}$	$\overline{18}$	$\overline{19}$	$\overline{11}$	$\overline{05}$	$\overline{03}$	$\overline{04}$	13	12	08	04	$\overline{03}$	02	$\overline{05}$	$\overline{11}$	$\overline{16}$	$\overline{10}$
10	17	23	23	18	07	02	$\overline{04}$	$\overline{11}$	$\overline{12}$	$\overline{07}$	$\overline{11}$	$\overline{10}$	12	21	27	20	06	03	04	08	11	07
$\overline{09}$	02	16	20	15	08	$\overline{01}$	$\overline{04}$	01	02	$\overline{04}$	$\overline{07}$	$\overline{05}$	$\overline{09}$	00	11	05	$\overline{03}$	$\overline{03}$	04	04	$\overline{01}$	$\overline{04}$
02	03	17	26	17	02	$\overline{05}$	$\overline{04}$	$\overline{02}$	$\overline{08}$	$\overline{08}$	$\overline{01}$	04	$\overline{02}$	05	08	05	00	$\overline{01}$	$\overline{04}$	$\overline{12}$	$\overline{05}$	$\overline{04}$
08	11	12	13	13	09	04	$\overline{02}$	$\overline{02}$	03	$\overline{04}$	04	09	10	08	09	11	02	07	04	02	$\overline{03}$	$\overline{08}$
03	21	39	46	36	25	09	$\overline{08}$	$\overline{18}$	$\overline{16}$	$\overline{14}$	$\overline{11}$	$\overline{04}$	17	30	37	38	22	10	22	31	30	18
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07	04	10	13	12	14	10	08	00	$\overline{07}$	$\overline{09}$	$\overline{08}$	$\overline{04}$	11	01	04	11	06	14	11	09	12	15
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13	11	11	11	12	10	03	$\overline{07}$	$\overline{08}$	$\overline{05}$	$\overline{06}$	$\overline{07}$	$\overline{03}$	08	12	14	11	09	09	00	$\overline{02}$	01	00
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03	26	49	55	47	33	12	$\overline{04}$	$\overline{17}$	$\overline{24}$	$\overline{22}$	$\overline{16}$	$\overline{09}$	20	38	48	44	34	11	30	41	41	25
02	15	34	44	32	14	01	$\overline{04}$	$\overline{03}$	$\overline{04}$	$\overline{01}$	05	12	08	22	28	22	05	04	13	15	10	03
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07	22	41	52	29	31	19	11	05	01	$\overline{04}$	$\overline{09}$	$\overline{19}$	12	24	32	29	17	00	12	16	14	08
09	15	19	20	21	14	08	02	$\overline{05}$	$\overline{02}$	00	$\overline{07}$	$\overline{18}$	00	00	09	04	$\overline{01}$	$\overline{10}$	$\overline{09}$	00	03	00
$\overline{06}$	$\overline{01}$	06	17	21	24	13	03	00	03	05	$\overline{07}$	$\overline{06}$	$\overline{06}$	$\overline{08}$	$\overline{04}$	07	14	$\overline{01}$	00	03	04	02
12	12	18	23	23	23	12	11	09	02	00	$\overline{04}$	$\overline{09}$	01	07	06	07	11	08	04	01	06	05
$\overline{06}$	$\overline{05}$	$\overline{01}$	02	06	05	07	03	00	00	$\overline{06}$	$\overline{13}$	$\overline{13}$	$\overline{04}$	00	00	$\overline{01}$	04	$\overline{01}$	03	14	09	11
14	14	08	04	04	02	$\overline{06}$	$\overline{02}$	03	$\overline{03}$	$\overline{16}$	$\overline{18}$	$\overline{17}$	02	$\overline{06}$	$\overline{01}$	03	04	09	05	07	05	06
$\overline{01}$	03	20	29	20	05	$\overline{02}$	04	06	04	01	$\overline{09}$	$\overline{10}$	14	23	24	15	06	16	17	16	09	03
$\overline{04}$	08	13	20	17	13	03	$\overline{02}$	$\overline{10}$	$\overline{09}$	$\overline{05}$	$\overline{03}$	01	06	16	21	22	19	06	09	07	10	14
$\overline{03}$	12	23	29	21	04	$\overline{12}$	$\overline{16}$	$\overline{19}$	$\overline{12}$	$\overline{04}$	$\overline{01}$	08	02	13	22	13	$\overline{01}$	$\overline{02}$	01	06	01	$\overline{09}$
$\overline{03}$	11	31	31	21	09	11	07	$\overline{05}$	$\overline{14}$	$\overline{17}$	$\overline{21}$	$\overline{12}$	$\overline{01}$	09	12	10	13	$\overline{09}$	01	$\overline{01}$	$\overline{02}$	04
15	27	42	48	43	28	18	10	01	$\overline{05}$	$\overline{12}$	$\overline{19}$	$\overline{21}$	27	36	35	36	31	23	26	28	27	26
18	24	24	27	27	20	11	02	$\overline{07}$	$\overline{01}$	$\overline{03}$	$\overline{10}$	$\overline{12}$	20	21	18	19	17	13	16	16	14	11

machines. The correlation coefficients for 1-, 2-, 3-, ... 36-day intervals (with the exception of 18- and 22-) and for 52-, 53-, ... 56- and 79-, 80-, ... 83-day intervals for each year 1890 to 1944 are listed in Table 1. Negative coefficients are overscored and all coefficients for convenience are multiplied by 100. Since the correlation coefficients were computed for several intervals simultaneously, the "year" for which recurrence was measured does not always correspond to the calendar year, the difference being as much as 35 days in addition to the length of the interval. The initial days of the 27-, 54-, and 8-day intervals, however, comprise a calendary year. The errors thus introduced are unimportant since the coefficients will not be treated separately.

The characteristics of the recurrence tendency of magnetic activity are well demonstrated by these auto-correlation coefficients. Figure 1 shows the variation in the strength of the tendency during the 55 years, represented by the annual coefficients for the 27-day interval; these are the largest in the first recurrence pulse in more than 80 per cent of the individual years. The cyclic variation of the recurrence tendency is more apparent if this curve is smoothed according to $r_0 = (r_{-1} + 2r_0 + r_{+1})/4$ (where the subscripts refer to years) as in Figure 2A. We recall that the division of the data into approximately calendar years was arbitrary. The recurrence cycle is approximately 180 degrees out of phase with the curve of sunspot numbers (see Fig. 2B) and also with the curve of the μ-measure of magnetic activity (Fig. 2C), but evidently closely related to both. The recurrence cycle and the annual percentage of low latitude sunspots ($\leq 10°$) compiled from the Greenwich publications, are remarkably similar. The correlation between the unsmoothed 27-day recurrence and the others is r = -0.56, -0.40, and +0.52, respectively.

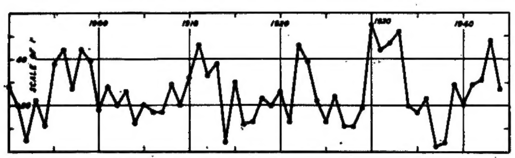

Fig. 1--27-day recurrence correlation coefficients, 1890-1944

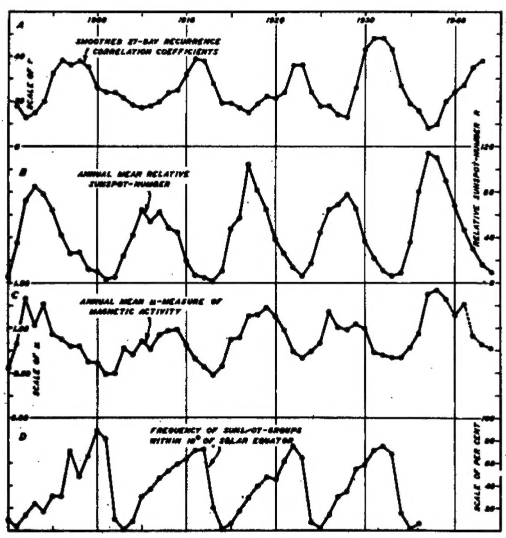

Fig. 2--Comparison of data relative to 27-day recurrence tendency, 1890-1944

To study variation in strength of recurrence during a single solar cycle, we may consider the average cycle to be made up of three years between minimum and maximum year and six between maximum and minimum, typical for the last eight sunspot cycles. Averaging the 27-day recurrence coefficients for the maximum and minimum years and distributing the coefficients of the remainder of the years proportionately according to this scheme, we obtain Figure 3, which shows that recurrence is most pronounced about a year before minimum. It is generally strong from about two years after maximum through the minimum year, and generally weak during the remainder of the cycle. For comparison, the annual sunspot numbers are also shown in Figure 3 for the same period, combined in the same way.

The variation from year to year of the smoothed coefficients for 54- and 81-day intervals follows closely that for the 27-day interval (see Fig. 4). The relative magnitudes of the 0-, 27-, 54-, and 81-day recurrence pulses for the stages strong and weak recurrence are shown in Figure 5.

Existence of any significant recurrence intervals other than about 27-days or multiples thereof should be detectable in the results given in Table 1. The annual coefficients for all intervals are averaged in Figure 6 for four periods of sunspot activity . . . minimum (calendar year of epoch ± one year), maximum, and the intervening ascending and descending years of the sunspot cycle. The very strong maximum in the coefficients at about 27 days is clearly shown, but no other pronounced maximum is revealed. In the descending and minimum periods of solar activity the coefficients for 13-, 14-, and 15-day intervals are slightly larger than on adjacent days, about zero instead of slightly negative. Reference to Table 1 shows that in a few individual years, for instance 1895, 1922, 1930, and 1942, the coefficients were higher than average at about these intervals. Possibly this illustrates a tendency for active solar regions to be situated 180 degrees apart in longitude at these stages of the solar cycle. A few coefficients of distinctly above average

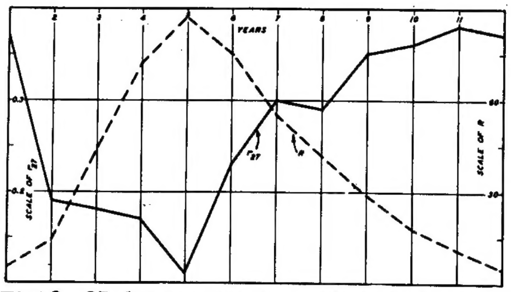

Fig. 3--27-day recurrence coefficients (r_{27}) and sunspot numbers (R) for a typical 11-year cycle

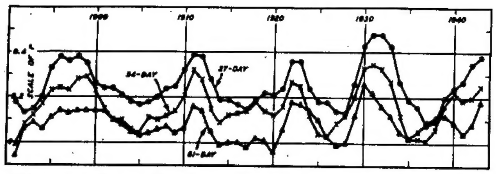

Fig. 4--Smoothed recurrence coefficients for 27-, 54-, and 81-day intervals, 1890-1944

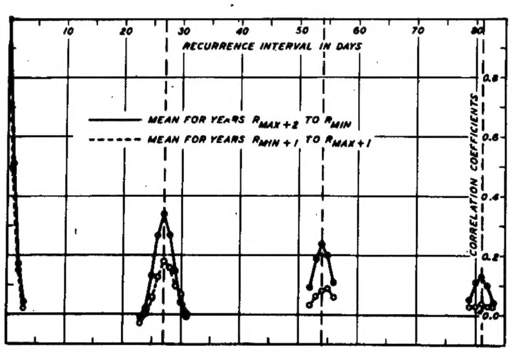

Fig. 5--First, second, and third recurrence pulses in years of strong and weak recurrence correlation

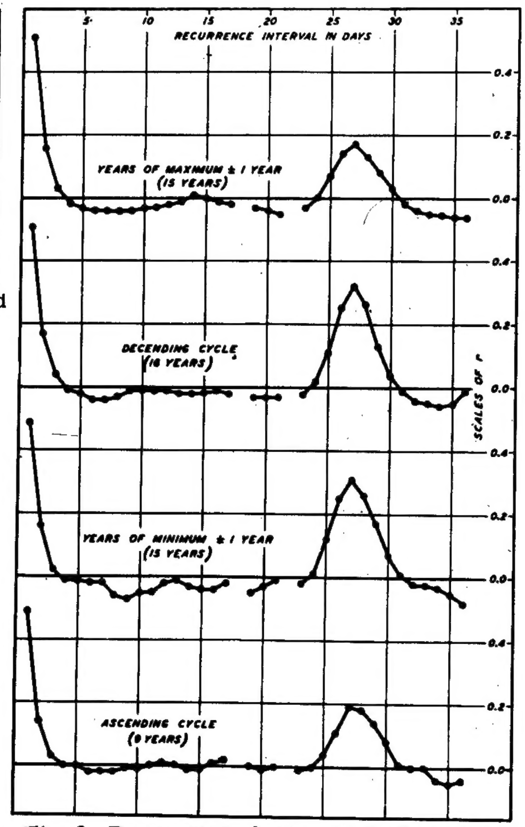

Fig. 6--Recurrence of magnetic C-figures at four stages of solar cycle; average of yearly correlation coefficients

value for intervals other than those of recognized recurrence appear in Table 1, such as for the six-day interval in 1900, the 20-day interval in 1911, and the 17-day interval in 1931. The evidence for consistent recurrence intervals other than those of about 27 days or multiples thereof is very weak; certainly no other interval is a rival in strength and prominence.

Consideration of the one-, two-, and three-day recurrence coefficients (see Table 1 and Fig. 6), shows that there is a very high correlation between the magnetic activity of one day and the first following, a much lower correlation with the second and very little with the third. Thus, the time unit of geomagnetic activity would seem to be on the average two to three days. Because intervals are considered, the initial part of the curve of Figure 6 is not doubled back on itself in order to find the average length of a disturbance. The two- to three-day geomagnetic time unit suggests, for instance, that forecasts of magnetic activity should not be made for daily intervals; preferably three-day average character figure estimates should be attempted.

Application to prediction--The correlation coefficients discussed in the preceding section may be applied to the prediction of magnetic character figures, in particular to the prediction of magnetic storms and the closely allied ionospheric disturbances. The problem is to predict a character figure, $x'_0 = ax_1 + bx_2 + \ldots$, for the value x_0, from the character figures of several previous days, $x_1, x_2 \ldots$, such that the squares of the errors of a large number of predictions, $\Sigma(x_0 - x'_0)^2$, is a minimum. Substituting and differentiating with respect to the unknown constants, a, b, . . ., we have as the required conditions

$$\Sigma(x_0x_1 - ax_1^2 - bx_1x_2 - cx_1x_3 - \ldots) = 0$$
$$\Sigma(x_0x_2 - ax_1x_2 - bx_2^2 - cx_2x_3 - \ldots) = 0 \text{ etc.}$$

If the character figures, x, are measured in terms of their standard deviations, then $\Sigma x_1^2/N = \Sigma x_2^2/N = \ldots = 1$. Likewise $\Sigma x_0 x_1/N$ is the auto-correlation coefficient, $r_{0,1}$, between x_0 and x_1; similarily $\Sigma x_0 x_2/N$ is $r_{0,2}$, etc. Given the correlation coefficients, we may solve the simultaneous equations for a, b, . . ., the constants for the least squares prediction formula $x_0' = ax_1 + bx_2 + \ldots$

Theoretically any number of terms may be included in a prediction formula. The accuracy of the prediction is not necessarily reduced by the inclusion of terms with small coefficients, or, in other words, terms based on weak correlation. The labor of solving a large number of simultaneous equations, however, is considerable, and so we will confine ourselves to formulas of five terms or less, including the more significant recurrence intervals of 1, 26, 27, 28, and 54 days.

The prediction method may also be extended to include nonlinear terms, for instance ones involving the consistency of the 27-day recurrence sequence of which the predicted day is a member. Thus, if the character figures on both the 27 and 54 preceding days were small, one might expect a greater probability that the character of the "zero" day will also be small. Accordingly disturbance indices $C_\alpha = \sqrt{(C_{-27} \times C_{-54})}$ and $C_\beta = \sqrt{(C_{-27} \times C_{-81})}$ were derived for each day, and $r_{0,\alpha}$, $r_{0,\beta}$ and the appropriate cross-correlation coefficients were computed so that terms involving x_α and x_β could be included in prediction formulas with x_{26}, x_{27}, and so forth.

Because of the demonstrated variation in the strength of recurrence with the solar cycle, separate prediction formulas are given for the stages of strong and weak recurrence. The average auto-correlation coefficients for the significant recurrence-intervals and $r_{0,\alpha}$ and $r_{0,\beta}$ are given for each subdivision of the cycle in Table 2. With these and the required cross-correlation coefficients, prediction formulas applicable to years of strong (1) and weak (2) recurrence are:

$$x'_0 = 0.23\bar{x} + 0.44x_1 + 0.11x_{26} + 0.12x_{27} + 0.02x_{28} + 0.08x_{54} \qquad \text{(A1)}$$

$$x'_0 = 0.33\bar{x} + 0.47x_1 + 0.06x_{26} + 0.07x_{27} + 0.03x_{28} + 0.04x_{54} \qquad \text{(A2)}$$

$$x'_0 = 0.46\bar{x} + 0.13x_{26} + 0.17x_{27} + 0.13x_{28} + 0.11x_{54} \qquad \text{(B1)}$$

$$x'_0 = 0.71\bar{x} + 0.06x_{26} + 0.09x_{27} + 0.10x_{28} + 0.04x_{54} \qquad \text{(B2)}$$

$$x'_0 = 0.49\bar{x} + 0.14x_{26} + 0.07x_{27} + 0.13x_{28} + 0.17x_\alpha \qquad \text{(C1)}$$

Table 2--Average auto-correlation coefficients for significant intervals in years of strong and weak recurrence condition

Type of year	Recurrence interval in days							
	1	26	27	28	54	81	α	β
Strong recurrence	0.51	0.27	0.34	0.27	0.24	0.13	0.35	0.28
Weak recurrence	0.49	0.13	0.18	0.16	0.08	0.04	0.18	0.15

The subscripts refer, of course, to preceding days, and α is as defined above. The units of x are those of the magnetic character figures, since conversion from units of standard deviation is effected by the term in $\bar{x}$, the mean value of the character figures, whose coefficient is $1 - (a + b + \ldots)$.

In formula (C1) the effect of the strong 27-day recurrence correlation is shared by x_{27} and x_α. Because the non-linear term receives the greater weight, we may infer that our supposition is valid that the consistency of the 27-day recurrence sequence has importance in prediction. In a formula like (C1), but also including a term in x_β, the coefficient in that term proved to be slightly negative. The coefficient of $\bar{x}$, however, was the same as in formula (C1), so we may conclude that x_β, though closely correlated with x_0, was entirely dependent on x_{27} and x_α and therefore contributed nothing of its own to the prediction.

The usefulness as well as the limitations of this method of predicting magnetic character figures is illustrated by trial predictions in typical years of strong and weak recurrence. Correlation of observed character figures with the predictions made by formula (A1) for 1944 gave r = 0.56 (see Fig. 7), by formula (B1), r = 0.37, while for the same period the simple 27-day correlation coefficient, $r_{0,27} = 0.31$. Predictions by formula (B2) for 1945 compared with observed values gave r = 0.17, when $r_{0,27} = 0.14$. The prediction formulas work out better than simple 27-day predictions as expected, the improvement being obtained mainly by including in the prediction data

from the day first preceding the predicted one. Regression towards the mean limits the range of the predicted character figures such that the predictions for years of weak recurrence lack detail. Independent correlations of some magnitude used in the formulas will increase the range and accuracy of the predictions. It might be expected, for instance, that other relationships such as solar-geomagnetic correlations could be used for more reliable prediction formulas.

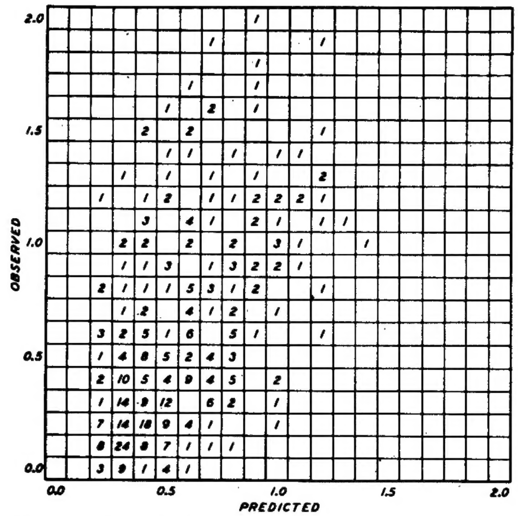

Fig. 7--Correlation diagram of magnetic character figures, 1944, observed and predicted by Formula A1; r = 0.56

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